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1989 J. Phys.: Condens. Matter 1 1285

(http://iopscience.iop.org/0953-8984/1/7/012)

J. Phys.: Condens. Matter 1 (1989) 1285–1298. Printed in the UK

Resistance of clean SNS sandwiches

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Received 23 September 1988

Abstract. Measurements are reported of the resistance of In/W/In sandwiches. The data close to T_c are found to fit the theory developed by Battersby and Waldram. In an attempt to explain further the data below T_c new expressions for the excitation reflection and transmission coefficients at the interface have been developed, by matching solutions to the Bogoliubov equations at the interface in two simple models. These models give poor fits to the data at temperatures below about 0.6 T_c, where the theoretical curve is found to have the wrong curvature. This discrepancy has been resolved by allowing for the penetration of the energy gap of the In into the W.

1. Introduction

The behaviour of the resistance of superconductor–normal–superconductor (SNS) sandwiches when the superconductor is clean and close to its transition temperature is faily well understood. Pippard *et al* (1971) first obtained adequate agreement between experiment and theory. Their theory was later extended to include a more realistic physical model by Waldram (1975). By using simplified boundary conditions in this theory, Battersby and Waldram (1984) again obtained agreement with experiment close to T_c. The purpose of this paper is to describe a new experimental investigation of the properties of clean SNS sandwiches and a more realistic form of the Waldram theory which describes their resistance down to temperatures well below T_c.

Figure 1 shows typical $R_{SNS}(T)$ data obtained with clean superconductors. In general the resistance is slowly varying up to about 0.8 T_c while above that temperature it begins to rise and diverges at T_c. As discussed by Waldram (1975) the divergence is caused by excitations with energy greater than Δ penetrating from N into S, which maintains a charge imbalance in S. As T_c is approached the rate of relaxation of charge imbalance falls to zero, leading to the observed divergence in the resistance at T_c. In this paper, however, we shall be primarily interested in the behaviour at temperatures below 0.8 T_c. In this range the resistance *falls* gradually with increasing temperature before reaching a minimum. At first sight we might expect that, at temperatures sufficiently low that all excitations arriving at the interface from N have energies much less than Δ, all excitations would be Andreev reflected with electron–hole inversion (Andreev 1964), and without boundary resistance. The reason why resistance is in fact seen is the subject of this paper. It is clear that there must be a mechanism which causes some excitations of energy less than Δ to be reflected normally, that is, without electron–hole inversion, which is probably related to an imperfection of the interface, either intrinsic between the Fermi

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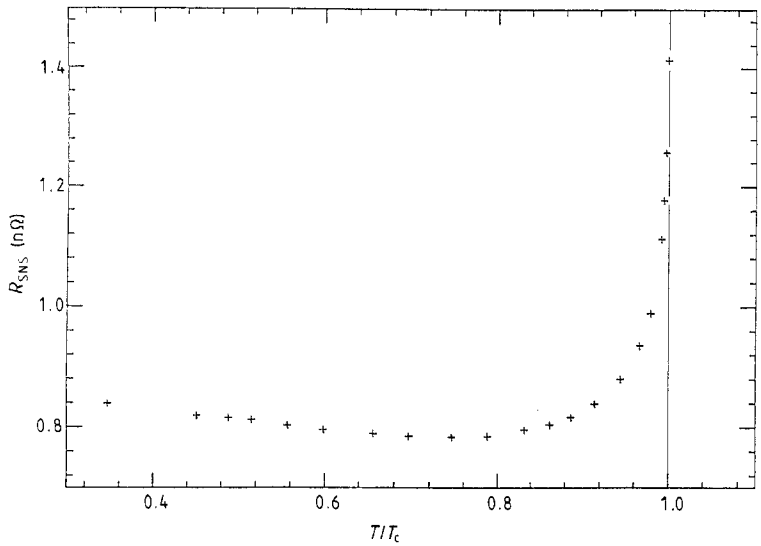

Figure 1. Typical $R_{SNS}(T)$ data obtained with clean In/W/In sandwiches.

wavefunctions of the two metals or some layer of excess scattering at the interface. Harding *et al* (1974) showed that the low-temperature resistance of SNS sandwiches was very sensitive to the sample making technique, which suggests that interface contamination can be important.

Harding *et al* describe a model which satisfactorily predicts the form of $R_{SNS}(T)$ in the range below 0.8 T_c. In this model part of the interface was taken to be covered with a thin layer of some insulator and the rest was 'perfect' NS contact (figure 2). The overall resistance was therefore that of the NIS contact (assumed to be the zero bias BCS tunnelling resistance) in parallel with that of the NS interface. The model may be questioned, however, firstly because in order to fit the experimental data Harding *et al* found it necessary to use values of Δ in Pb of about half the expected BCS value. They state that this could possibly be due to the oxide patches being of the order of a coherence length in size so that the energy gap is reduced by the area of NS contact. More recently, however, Shelankov (1985) has suggested that the unrealistic values required for the energy gap may result from the BCS tunnel theory being applied outside the temperature

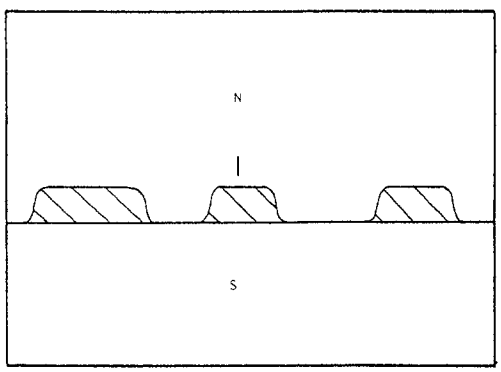

Figure 2. Schematic illustration of Harding's model of the NS interface.

range where it is valid. Secondly, it was not clear why some parts of the interface should be perfectly clean when others had a significant insulating layer. A model which has some form of imperfection over the entire area of the interface would seem more natural. Such a model is described in this paper.

2. Experimental method

The system used for the present work was In/W/In, in contrast to most of the previous work which used Pb/Cu/Pb sandwiches. This work was the first to use In as the superconductor which means that it was an important test of the theory using a weak coupling superconductor. The use of W also made possible the use of a new method of manufacturing clean SN contacts. This sample making technique has been described in detail in Lean (1987) and Lean and Waldram (1989). In brief, the interfaces were formed by resistively heating a slice of W in a high vacuum with pieces of In on top. The In melted and coated the surface of the W at about orange heat at which temperature surface contaminants are driven off the W. The interfaces were therefore formed while the surface of the W was relatively free of surface contamination. After cooling the vacuum was broken, the W slice inverted and the procedure repeated to form the second NS interface. The resulting W with a layer of In on both surfaces was then converted to a sample with well defined interface area by a simple procedure involving the use of Pyrex tubes to form the casts.

The resistance properties of these sample were measured in the range 1.2 to 4 K using a SQUID voltmeter in a cryostat described by Battersby and Waldram (1988). The quality of the NS interfaces, as judged by the fall in resistance between about 0.3 T_c and the minimum value, was about the same as those used by Harding (1973) and Battersby (1981).

3. Comparison of data with the Battersby theory

In this section the simple boundary conditions used by Battersby are discussed and the data are compared with the resultant theory. We do so because it is of interest to compare this simplified theory with data on a weak coupling superconductor. It also serves to introduce certain difficulties with the simplified theory which motivated the more complex models described in the rest of this paper.

The general result used in this work is that of Waldram (1975) for the boundary resistance of an NS interface, expressed in terms of integrals over energy of various combinations of reflection and transmission coefficients (his equations 3.14–3.16). The coefficients concerned are the probabilities of normal and Andreev reflection and normal and Andreev transmission of excitations of energy E denoted $R_N(E)$, $R_A(E)$, $T_N(E)$, $T_A(E)$ respectively. The theories described in this paper are concerned with developing realistic energy dependent expressions for these quantities.

The simplified boundary conditions used by Battersby were as follows.

(i) $R_N = 0$ for all energies, which assumed there is no oxide etc or mismatch at the interface to scatter back the excitations 'normally'. This might be a reasonable approximation for some pairs of metals if the superconductor is clean and if care is taken to prepare very clean interfaces.

(ii) $R_A = 1$ below the gap, which follows from (i) since no transmission is possible.

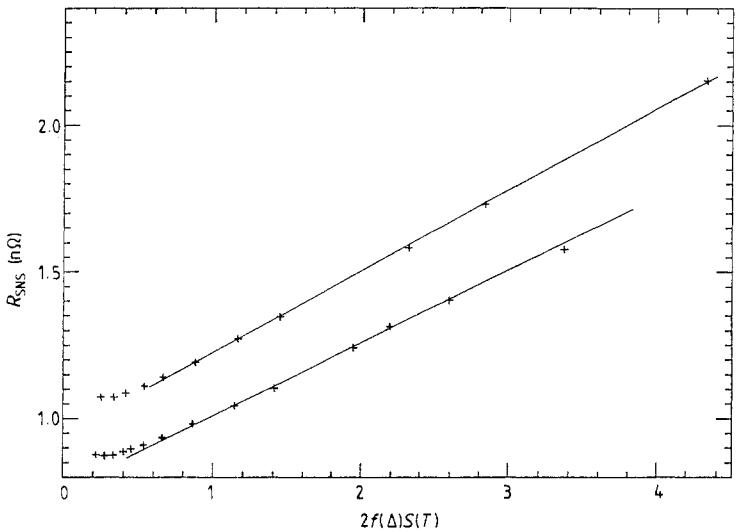

Figure 3. Typical R_{SNS} data for two samples plotted to show fit to Battersby theory.

(iii) $R_A = 0$ above the gap. Andreev (1964) showed that there is an appreciable probability of Andreev reflection for excitations with energy just above Δ. However, this is expected to have a negligible effect near T_c where most excitations will have energies many times Δ and a low Andreev reflection probability.

It is emphasised that because of (iii) this theory was expected to be valid only close to T_c. Because of (i) all boundary resistance in this model is associated with charge imbalance in S.

Substituting these boundary conditions into Waldram's integrals and neglecting terms which turn out to be small leads to the result quoted by Battersby and Waldram (using slightly different notation)

$$Q_i = 2f(\Delta)(\lambda_3/l_0)^S. \tag{3.1}$$

Here, following Pippard *et al* (1971), the resistance is expressed by the dimensionless quantity Q_i which is the number of mean free paths in N required to give the same resistance. The resistance of the interface, R_i, is given by $R_i = Q_i(\rho l_0)^N/A$ where A is the area of the interface and l_0 is the excitation mean free path. (Throughout this paper superscript N or S implies that the quantity concerned is to be taken in N or S.) λ_3 is the diffusion length for branch crossing processes defined by Waldram as $(l_0 l_3)^{1/2}$ where l_3 is the mean free path for branch crossing processes.

This result may be given a simple interpretation. The resistance behaves as though the fraction $2f(\Delta)$ of excitations above the gap were flowing through a layer of S of thickness λ which had turned normal. In order to compare this theory with the data it is necessary to insert a specific form for $\lambda_3(T)$. The form $\lambda_3(T) = \lambda^0 S(T)$ used in this work was that derived by Battersby and Waldram (1989) (their equation 6.8). The fit to this theory is best seen by plotting R_{SNS} against $2f(\Delta)S(T)$. The data from two samples plotted in this way are shown in figure 3, and it can be seen that good straight lines are observed close to T_c with, however, deviation from the theory at low temperatures. Since l_0 varied significantly from sample to sample, the values of λ obtained from the gradients are best considered in terms of $l_3^0 = (\lambda^0)^2/l_0$. The value of l_3 should be constant

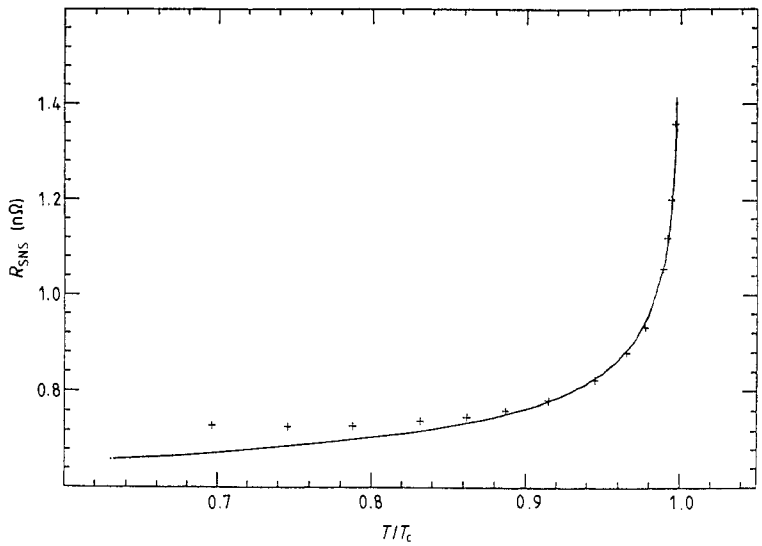

Figure 4. Typical R_{SNS} data with Battersby theory curve to show discrepancy at lower temperatures.

from sample to sample if the theory is obeyed. The values for four samples are shown in table 1. It can be seen that the values of l_3^0 are indeed fairly constant.

Figure 4 shows typical experimental data compared with the theoretical fit plotted with a linear temperature axis. It can be seen that the agreement is good above about $0.9\ T_c$. Below this the experimental data reaches a minimum before rising slowly as the temperature is reduced. In contrast the theoretical curve continues to fall as the temperature is reduced; as mentioned above this is due to the assumption $R_N = 0$ which means that subgap excitations do not contribute any interface resistance. The fact that extra interface resistance is seen experimentally at low temperatures is a clear indication that is necessary to insert more realistic boundary conditions in which $R_N \neq 0$ to model this behaviour. Such theories are described in the rest of this paper.

4. Step-function potential theory compared with experiment

The approach adopted to calculating more realistic boundary conditions was that of matching bulk solutions to the Bolgoliubov equations for the excitation state functions on the two sides of the interface. It is clearly necessary to choose some spatially varying potential to cause some normal reflection. Two idealised forms of potential have been used, the step-function potential described in this section and the delta function form described in the next.

The step-function potential was of the form shown in figure 5; both Δ and V are assumed to have steps at the interface. This model is intended to represent the effects of normal scattering caused by intrinsic mismatch between the bulk properties of the two metals rather than from any dirt etc at the interface. The stages in the derivation of the boundary conditions are given below.

(i) The bulk solutions $u(x)$, $v(x)$ to the Bogoliubov equations in N and S are written down. Below the gap in S they are very slightly evanescent (as described by Harding *et*

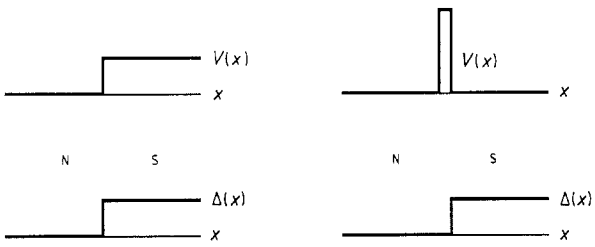

Figure 5. Potentials used in the models of the NS interface.

al 1973); clearly only the solutions which decay into S are physically meaningful in this situation. Above the gap the solutions represent the usual BCS excitation spectrum.

(ii) The total wavefunction is written down on each side of the interface. The incoming electron wave is assigned amplitude unity. The normal and Andreev reflected waves are assigned amplitudes r_N and r_A respectively. The transmitted waves are similarly assigned amplitudes t_N and t_A above the gap. Below the gap the transmitted evanescent modes are assigned amplitudes t_{A+} and t_B, following the notation of Harding *et al.*

(iii) $u(x)$, $v(x)$ and their slopes are set to be continuous at the interface. This leads to four equations both above and below the gap.

(iv) These equations are solved to give expressions for r_A, r_N, t_A, t_N in terms of E and $\gamma = k_{FS}/k_{FN}$, the ratio of Fermi wavevectors in the two metals.

(v) The amplitude reflection and transmission factors thus obtained are converted to flux amplitudes, denoted by R_A, R_N, T_A, T_N, by multiplying them by their complex conjugates. In the case of the transmission factors they also have to be multiplied by $\gamma v_S/v_N$ where the vs denote excitation velocities.

The results obtained are given below. For $E > \Delta$

$$\begin{aligned} R_N &= (1-\gamma^2)^2(\varepsilon/E)^2/D & T_N &= 2\gamma(1+\gamma)^2(1+\varepsilon/E)(\varepsilon/E)/D \\ R_A &= 4\gamma^2(1+\varepsilon/E)(1-\varepsilon/E)/D & T_A &= 2\gamma(1-\gamma)^2(1-\varepsilon/E)(\varepsilon/E)/D \end{aligned} \tag{4.1}$$

where $D = [u_B^2(1+\gamma)^2 - v_B^2(1-\gamma)^2]^2$. For $E < \Delta$

$$R_A = \{[1+(A-1)][1-(E/\Delta)^2]\}^{-1} \qquad R_N = 1 - R_A \tag{4.2}$$

where $A = (1+\gamma^2)^2/4\gamma^2$.

Several points about these results are worth mentioning.

(i) The particle conservation relations, $R_A + R_N + T_A + T_N = 1$ above the gap and $R_A + R_N = 1$ below it are obeyed.

(ii) The relation $R_A R_N = T_A T_N$ derived for supergap excitations in Appendix A of Battersby and Waldram (1988) by assuming electron–hole symmetry, is obeyed above the gap. Below the gap where $T_A = T_N = 0$ it breaks down.

(iii) If we set $\gamma = 1$ (i.e. no Fermi surface mismatch) the expected results are recovered: $R_A = 1$ below the gap, $R_A(E)$ has form of Andreev (1964) above the gap and $R_N = 0$ at all energies.

(iv) For $\gamma \neq 1$ (4.2) implies that $R_A = 1$ for $E = \Delta$ but decreases as E is reduced. $R_A(E)$ is plotted for several values of γ in figure 6. This is important since it provides a mechanism for the fall in resistance observed between 0.3 and 0.8 T_c: as the temperature is increased in this range the average energy of the excitations increases and there are therefore more excitations with lower values of R_N.

Before these boundary conditions could be used in Waldram's integrals to calculate

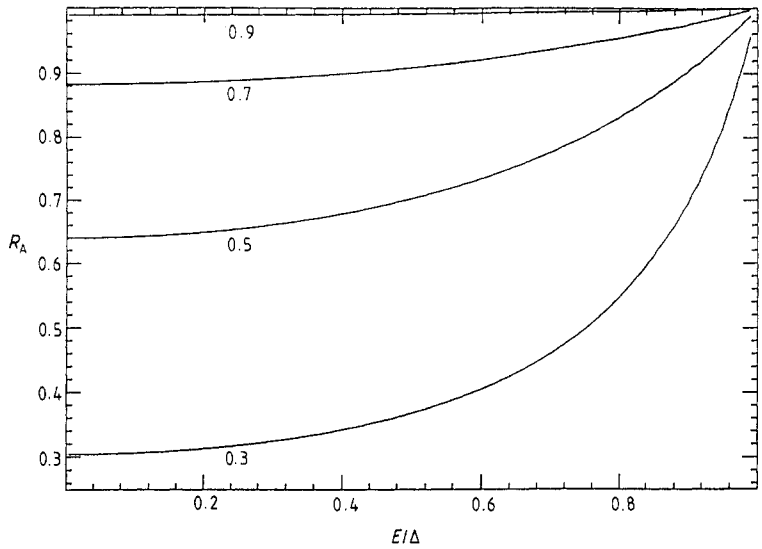

Figure 6. The Andreev reflection factor $R_A(E)$ for sub-gap electrons for various values of the mismatch parameter γ calculated from the step function model.

the boundary resistance the values of the other parameters of his theory had to be estimated. These were l_2 in W and In, defined by Waldram as the mean free path for inelastic processes which, as pointed out by Pippard *et al* (1971), can be taken as the mean free path for thermal conduction in ideally pure material. Estimates of l_2 were therefore obtained from thermal conductivity data of Guénault (1961) in the case of In and Wagner *et al* (1971) in the case of W. In the case of W, l_2 (the diffusion length for inelastic scattering) turns out to be rather longer than the thickness of the W slice, d (about 0.4 mm). In this case, as shown by Pippard *et al* (1971), l_2 should be taken as approximately $d/2$. For the In the value of l_2 inserted in the theory was temperature dependent (l_2 in fact increases rapidly with decreasing temperature, reflecting the reduction in the number of phonons available to take part in inelastic processes).

Other quantities were fitted as adjustable parameters. The value of l_3 was handled as in § 3: the temperature dependence of Battersby and Waldram was used and l_3^0 was left as an adjustable parameter. The other adjustable parameters in the theory were γ adjusted so that the fall in resistance at low temperatures was of the correct magnitude, T_c, adjusted by a few mK to fit the divergence below T_c as in § 3 and R_W, the constant resistance to be subtracted from the experimental curve to bring it into coincidence with the theory. R_W represents the bulk resistance of the W plus any interface resistance due to effects other than resistance between the two metals and was assumed to be temperature independent.

Table 1. Parameters fitted to the Battersby boundary conditions.

Sample no	l_0 (10^{-5} m)	l_3^0 (10^{-5} m)
91	2.0 ± 0.1	2.5 ± 0.3
97	1.2 ± 0.1	2.0 ± 0.2
99	1.6 ± 0.1	2.0 ± 0.2
108	0.8 ± 0.1	2.0 ± 0.3

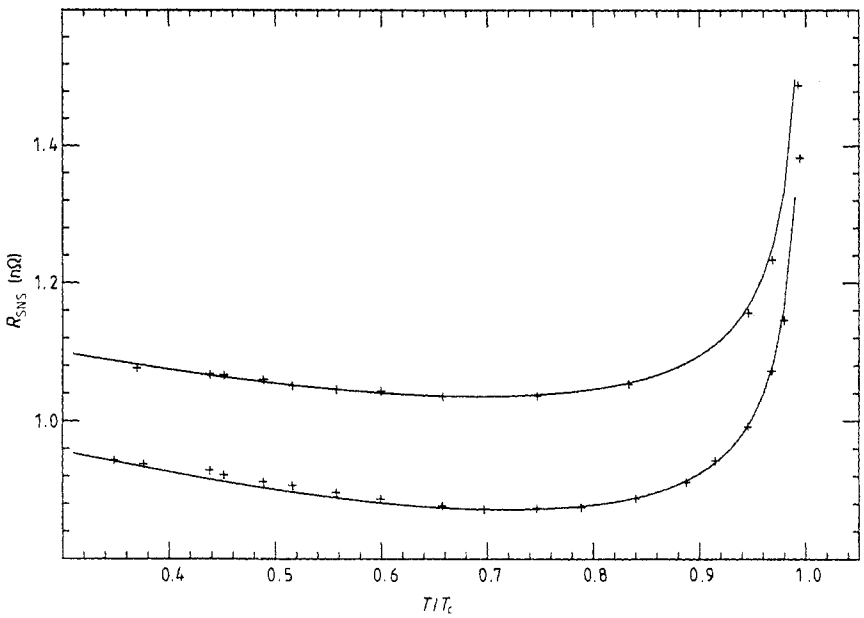

Figure 7. Fits of experimental $R_{SNS}(T)$ to step function model. The upper plot has been raised by 0.1 nΩ.

Once estimates of the various parameters had been made, the theoretical resistance curves were calculated from Waldram's integrals with the boundary conditions (4.1) and (4.2). The integrals were evaluated numerically. Typical fits to the experimental data with this theory are given in figure 7. It can be seen that the theory fits well from the minimum in the curve up to T_c. At low temperatures, however, the fit is not very good. The experimental curve shows signs of levelling out as the temperature is reduced below about 0.6 T_c whereas the theoretical fit has the opposite curvature. The reason for this poorness of fit at low temperatures is discussed in § 6. The values of l_3^0 and γ found for the best fits to the data for four samples are given in table 2. The relatively large errors quoted for l_3^0 in this case reflect the fact that the magnitude of the divergence at T_c is only a fairly slow function of l_3: the magnitude of the divergence is proportional to l_3 and hence to $\sqrt{l_3^0}$. The values of l_3 are again found to be fairly constant from sample to sample. In this case, however, the values are about a factor of six higher than those found using the Battersby boundary conditions described in § 3. It is easy to see the physical reason for this difference. In the step-function potential model there is a finite R_N which means that fewer supergap excitations penetrate into S. Just below T_c, where charge inbalance in S is the dominant source of resistance, the only way in which the

Table 2. Parameters fitted to step function boundary conditions.

Sample no	l_3^0 (10^{-4} m)	γ
91	1.4 ± 0.1	0.315
97	1.0 ± 0.1	0.305
98	1.4 ± 0.1	0.325
99	1.4 ± 0.1	0.325

theory can generate the same observed voltage is by assuming that l_3^0 is longer: in this way the same Q can be set up at the interface even though fewer excitations are entering S.

The main criticism which can be made of this model is that the step function potential implies that normal reflection is caused by the ratio, γ, of the k_F values, an intrinsic property of the two metals. As mentioned earlier, previous workers have found that the low temperature fall in the resistance is critically dependent on the interface preparation technique. This, together with the variation in the values of γ required to fit the present samples implies that the normal reflection is, in part at least, caused by imperfections at the interface rather than to mismatch between the two metals. (It should be noted that although γ only varies between 0.305 and 0.325, the low temperature fall is particularly sensitive to its value: this range corresponds to low-temperature resistances falling in the range 0.041–0.07 nΩ.) In order to overcome this criticism the delta function model was tried and is described in the next section.

5. Delta-function theory compared with experiment

The potential used in this theory was a delta function of the form $V = f\delta(x)$ with Δ again assumed to have a step at $x = 0$ (figure 5). This is intended to represent normal scattering caused by dirt etc at the interface. The real situation will probably be somewhere between this model and the one described in § 4, so comparison is of interest. The derivation of the boundary conditions in this case was carried out in the same way as in the mismatch theory. The results derived from this theory are given below. For $E > \Delta$

$$\begin{aligned} R_N &= F^{-1}(a^4 + 4a^2) & T_A &= F^{-1}[2a^2[2a(1 - \varepsilon/E)(E/\varepsilon)] \\ R_A &= F^{-1}[16b(1-b)^{-2}] & T_N &= F^{-1}[2(4 + a^2)(1 + \varepsilon/E)(E/\varepsilon)] \end{aligned} \tag{5.1}$$

where

$$F = [4(1-b)^{-1} + a^2]^2 \qquad a = 2mf/\hbar^2 k_F$$
$$b = (v_B/u_B) = (1 + \varepsilon/E)(1 - \varepsilon/E).$$

For $E < \Delta$

$$R_A = [1 + (4R/(1-R))(1 - (E/\Delta)^2)]^{-1} \qquad R_N = 1 - R_A \tag{5.2}$$

where $R = a^2/(4 + a^2)$.

The important point here is that the expression for R_A below the gap is of the same form as (4.2) which was obtained using the mismatch theory. In order to convert one to the other the parameter A involving γ must be replaced by R, involving a, according to

$$A - 1 = 4R/(1 - R). \tag{5.3}$$

This means that well below T_c the two theories will give the same results. In particular the fit to the fall in R_{SNS} from the lowest temperatures to the minimum will not be any better with this theory than it was with the mismatch theory. The expressions above the gap are different, however, so that differences may be expected close to T_c. The new expression for R_A still tends to that of Andreev in the case of a perfect interface where $a = 0$.

These boundary conditions were fitted to the data in the same way as the mismatch boundary conditions. Two examples of fits to experimental data using this theory are

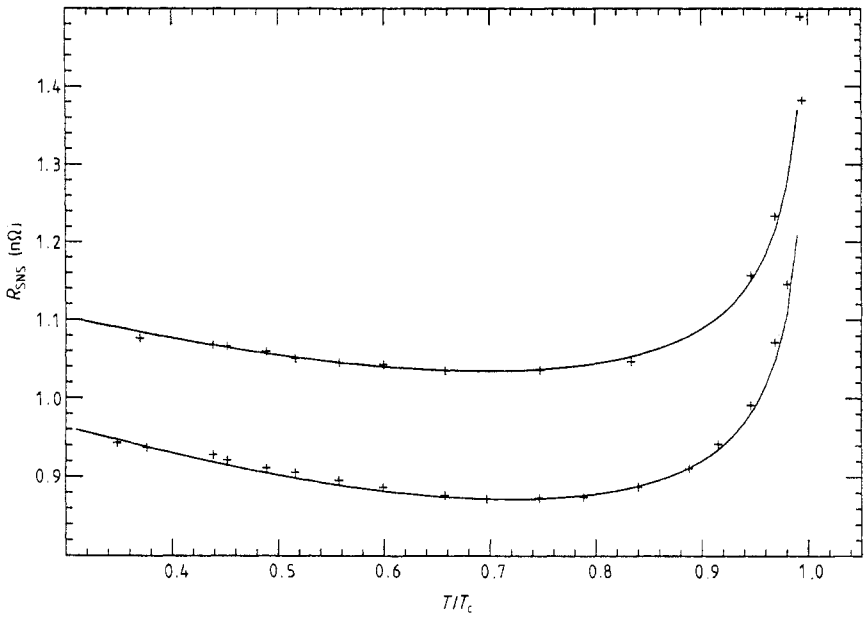

Figure 8. The data of figure 7 fitted to delta function model.

given in figure 8. It can be seen that again the fit below T_c is good and, as expected, the low-temperature fit is no better than before. The values of l_3^0 and a for the best fits are given in table 3. The values of l_3^0 are even greater than those required in the mismatch theory. They are now about a factor of 20 greater than in the simple theory discussed in § 3. However it should again be remembered that the physically important quantity (which is proportional to the resistance generated by the Q) is λ_3 which varies as $\sqrt{l_3^0}$. λ_3 in this case will, therefore, be only about a factor of 4.5 greater than in the simple case. The values of a obtained for the best fits are in all cases close to those implied by (5.3) and the fits to the mismatch theory (table 2). These results confirm that, as expected, this theory is essentially the same as the mismatch one at low temperatures. The reflection and transmission factors obtained above the gap are different, however, and this leads to differences in the magnitude of the divergence below T_c.

6. Theory including the proximity effect

A better fit to the low-temperature behaviour has been obtained by taking into account the effect of a non-zero Δ extending into N. This is likely to be significant in the present samples because W is a superconductor with an appreciable T_c (about 0.02 K). The likely

Table 3. Parameters fitted to δ-function boundary conditions.

Sample no	$l_3^0/10^{-4}$ m	a
91	4 ± 0.6	1.45
97	3 ± 0.6	1.53
98	4 ± 0.6	1.40
99	4 ± 0.6	1.40

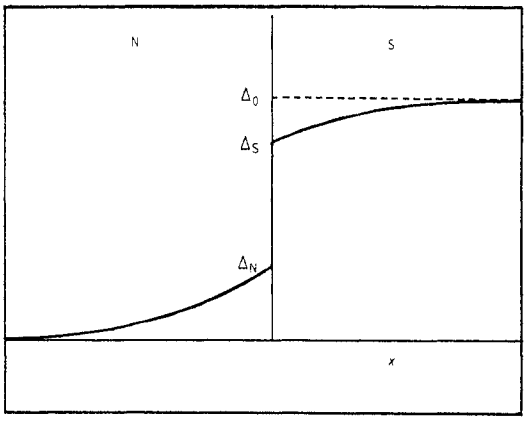

Figure 9. The form of $\Delta(x)$ near the interface assumed in the proximity effect calculation.

form of $\Delta(x)$ in the region of the interface is shown in figure 9. The form of Δ will differ from the step assumed so far in two ways. Firstly it will have a finite value just on the normal side of the interface (denoted Δ_N) and will decay into N. Secondly, on the S side close to the interface Δ will be pulled down from its bulk value Δ_0. This second effect will not directly affect the excitations in the temperature range of interest as they will mostly have $E \ll \Delta_0$; it will, however, affect the value of Δ_N.

The tail of Δ extending into N will mean that excitations in N with $E < \Delta_N$ impinging on the interface will be reflected inside N rather than at the interface itself. This will have the effect that the excitations will no longer interact directly with the impurities or mismatch at the interface. However, as discussed in Harding *et al* (1973), evanescent tails of the excitations will extend into the forbidden region and these will still interact with the potential step at the interface to produce a reduced resistance. In order to assess the size of this effect, it is necessary to know the relative sizes of K_N^{-1} the decay length of Δ in N as obtained from the theory of the proximity effect, and the decay lengths of the evanescent modes which are related to the length l_a introduced by Harding *et al*. If K_N were to be much shorter than the evanescent decay length (so that the evanescent waves hardly decay at all between the point of reflection and the interface) then the reduction in the resistance would be very small and this effect would be negligible.

K_N was calculated from the result of Hook and Waldram (1973) in the clean limit

$$K_N^{-1} = 0.42\,\hbar v_F/\pi k_B T. \tag{6.1}$$

Inserting values for In leads to the result $K_N^{-1}(T/K) = 2.5 \times 10^{-7}$ m, so at the lowest temperatures at which data was taken K_N^{-1} will be about 2×10^{-7} m. Estimates can be made of the evanescent decay length close to the interface, using $l_a = \hbar v_F/2\Delta$ (as shown by Harding *et al*) and realistic estimates of the size of Δ, which give typical values of about 5×10^{-7} m for zero-energy excitations. Thus the effect on R_N is expected to be significant.

In order to calculate the influence of the proximity effect on the resistance at low temperatures, we also need estimates of the parameters Δ_0/Δ_S and Δ_S/Δ_N which together give the value of Δ_N. The latter was calculated using the boundary condition derived by de Gennes (1964):

$$\Delta_s/\Delta_N = (N_0V)^S/(N_0V)^N. \tag{6.2}$$

Although Hook and Waldram have pointed out that this boundary condition is not very

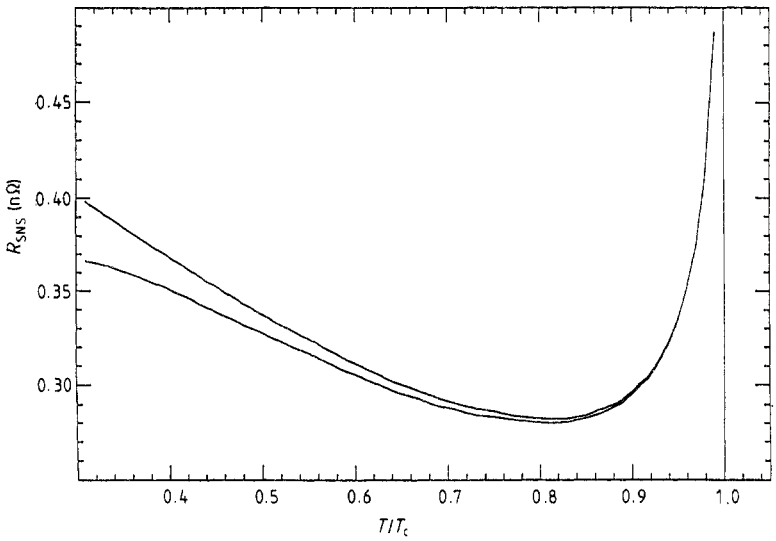

Figure 10. Delta function model theoretical curves for $a = 1.5$ with (lower curve) and without (upper curve) allowance for the proximity effect.

rigorous, it was used in this work since it only scaled an adjustable parameter in the final theory. N_0V was calculated for In and W from the values of T_c and the Debye temperature, using the BCS expression for T_c. It is found that the ratio Δ_S/Δ_N is expected to be about three. The ratio Δ_0/Δ_S which gives the extent that Δ is pulled down in S by the interface is harder to estimate. The theory of McMillan (1968) suggests it is approximately two. In view of these uncertainties, two values of the ratio $X = \Delta_0/\Delta_S$ were tried in the theory.

The delta-function potential theory described in the § 5 was modified to include this effect. This was done by using the results of Tomlinson (1973), who also performed this calculation. The supergap boundary conditions are, of course, unchanged. The subgap ones for when $E < N$ are modified, by inserting Tomlinson's form for R_N

$$R_N = \frac{4R(1 - E^2/\Delta_0^2)}{(\theta^2 - R/\theta^2) + 4R(1 - E^2/\Delta_0^2)} \tag{6.3}$$

where

$$\theta = \exp\left[\frac{K_N^{-1}\Delta_N}{\hbar v_F} f\left(\frac{E}{\Delta_N}\right)\right]$$

and $f(u) = (1 - u^2)^{1/2} - u\cos^{-1}u$.

This expression gives the same value of R_N as (5.2) when $E = \Delta_N$. It should be noted that the function θ represents the total decay of the evanescent wave amplitude between the point of Andreev reflection and interface. Figure 10 compares theoretical curves (for $a = 1.5$) with and without this effect. It can be seen that, as expected, the proximity effect significantly decreases the resistance at low temperatures but has little effect near T_c.

The modified theory was fitted to the experimental data. Two values of Δ_0/Δ_N were tried, 3 and 6. Typical fits are shown in figure 11. In order to balance the depression of the resistance at low temperatures it is found that the values of a required are slightly

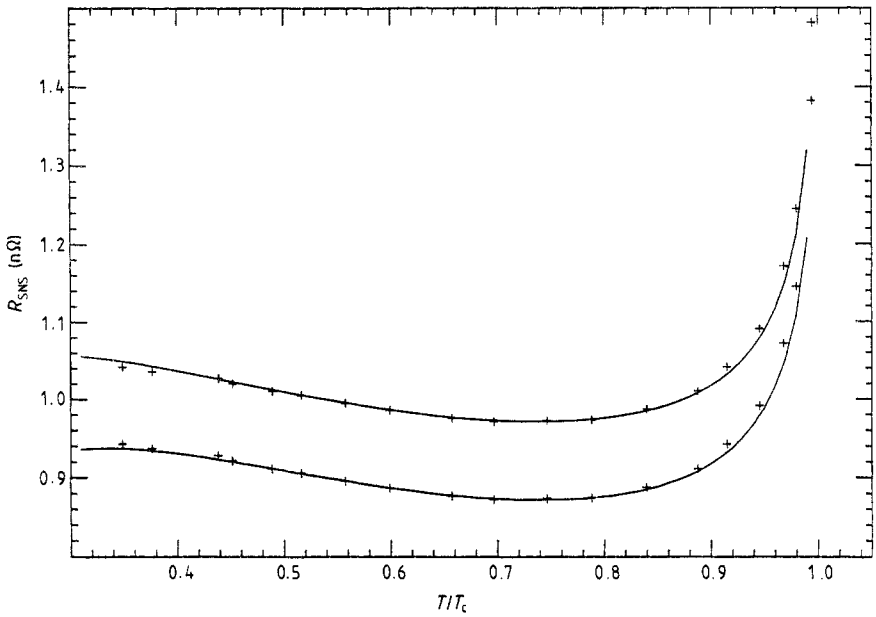

Figure 11. Typical fit to proximity effect theory with $\Delta_0/\Delta_N = 3$ (upper curve, raised by 0.1 $n\Omega$) and 6 (lower curve).

increased. It can be seen that the fits at low temperatures are much better than in the unmodified theory: the theoretical curves have the correct curvature at low temperatures. It turns out, rather surprisingly, that the $\Delta_0/\Delta_N = 3$ curves fit the data better than $\Delta_0/\Delta_N = 6$ ones. If (6.2) is taken seriously this implies that Δ is not pulled down significantly in S close to the interface.

It seems therefore, that the shape of the $R_{SNS}(T)$ curve in In/W/In at low temperatures is strongly influenced by the proximity effect. It would be interesting to extend measurements on this system to lower temperatures. One would expect that the resistance would be observed to pass through a maximum and begin to fall again.

7. Summary

We have reported measurements of NS interface using In/W/In sandwiches for the first time. The data were first compared with the simplified theory developed by Battersby and Waldram (1984) from that of Waldram (1975). It was found that agreement with the experimental data was good close to T_c but poor below about $0.8\,T_c$. This was to be expected since the behaviour at lower temperatures is dominated by normal reflection of excitations caused by dirt etc at the interface which is neglected in this theory.

Attempts were therefore made to improve on the simplified theory by calculating more realistic expressions for the interface reflection and transmission coefficients. In order to include some normal reflection two models were used for possible imperfections of the interface, a step-function form for the normal potential intended to model mismatch between the Fermi wavevectors of the two metals, and a delta-function potential intended to represent sources of scattering actually at the interface. These two models gave very similar results. It was found that agreement between experiment and the resulting theoretical curves was good in the range above about $0.6\,T_c$. At the lowest

temperatures at which data was taken (about 0.3 T_c), however, the theoretical curves were found to have the wrong curvature. This latter discrepancy was resolved by modifying the delta-function theory to include the effect of penetration of the In energy gap into the W. This caused the excitations to be reflected inside the W at some distance from the imperfect interface, thus reducing the observed resistance at low temperatures. Our experimental data fitted quite well to the resulting theoretical curve when reasonable estimates were made of the parameters in this theory.

Acknowledgments

We should like to thank A B Pippard for helpful discussions of the theory, S J Battersby for help with the measurement apparatus and T E Brown for providing us with high purity *W*. H W L was supported by a grant from the Science and Engineering Research Council.

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